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Electromagnetic Structure of Light Baryons in Lattice QCD

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Abstract

A method for computing electromagnetic properties of hadrons in lattice QCD is applied to the extraction of electromagnetic properties of the octet baryons. This allows a determination of the full dependence of the baryon masses on the charges and masses of the valence quarks. Results of a first numerical study (at $\beta = 5.7$ with Wilson action and light quark masses fixed from the pseudoscalar meson spectrum) are reported. The octet baryon isomultiplet splittings (with statistical errors) are found to be: $\mathrm{N} - \mathrm{P} = 1.55(\pm 0.56)$, $\Sigma^0 - \Sigma^+ = 2.47(\pm 0.39)$, $\Sigma^- - \Sigma^0 = 4.63(\pm 0.36)$ and $\Xi^- - \Xi^0 = 5.68(\pm 0.24)$ MeV. Estimates of the systematic corrections arising from finite volume and the quenched approximation are included in these results.

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The accurate determination of the neutron-proton mass difference represents one of the thorniest and longest standing problems of hadronic physics. An early review of Zee [1] concludes with 19 appendices outlining various approaches to the problem. The advent of modern quantum chromodynamics (QCD) has dissipated a great deal of the conceptual fog surrounding this problem, but we are still faced with the difficult technical problem of computing accurately the competing effects of the up-down quark mass difference (the "tadpole", in ancient jargon) and the photon cloud energy in a complicated hadronic state. Since the contribution to hadronic mass splittings within isomultiplets from virtual photon effects is comparable to the size of the up-down quark mass difference, an accurate computation of isospin splittings requires the inclusion of electromagnetic effects in the context of nonperturbative QCD dynamics. In this letter, we apply a method recently used [2] to extract the electromagnetic contributions to pseudoscalar masses to the problem of the octet baryon spectrum. In addition to the SU(3) color field, a U(1) electromagnetic field on the lattice is introduced and treated by Monte Carlo methods. The resulting SU(3)$\times$U(1) gauge configurations are then analyzed by standard hadron propagator techniques. One of the main results of our earlier work [2] was to demonstrate that calculations done at larger values of the quark electric charges (roughly 2 to 6 times physical values) lead to accurately measurable hadronic isospin splittings, while still allowing perturbative extrapolation to physical values. A useful result of that work was the extraction of up, down and strange quark bare lattice masses appropriate for a lattice of quenched configurations at β=5.7 (and Wilson action), which can serve as input to a computation of the baryonic isomultiplets.

The strategy of the present calculation is as follows. Quark propagators are generated in the presence of Coulomb gauge background SU(3)$\times$U(1) fields described above. (A detailed description of our formulation of the U(1) field is presented in our previous work[2].) Quark propagators are calculated for a variety of electric charges and light quark mass values, and with either a local or smeared source. 187 gauge configurations, separated by 1000 Monte Carlo sweeps, were generated at $\beta = 5.7$ on a $12^3 \times 24$ lattice. We have used four different values of charge given by e_q =0, -0.4, +0.8, and -1.2 in units in which the electron charge is $e = \sqrt{4\pi/137} = .3028\ldots$. For each quark charge we calculate propagators for three light quark mass values in order to allow a chiral extrapolation. From the resulting 12 quark propagators, 936 different octet baryon three-quark combinations can be formed. The baryon propagators are then computed and masses for all 936 states extracted.

The analysis then proceeds by a combination of chiral and QED perturbation theory. In quenched QCD it is known [3] that baryon masses are described by a function of the bare quark masses involving not only linear but also nonanalytic $m_q^{3/2}$ terms, as well as terms involving logarithms of the quark mass arising from the same hairpin diagrams familiar in the quenched meson spectrum [4, 5]. The latter terms arise from the hairpin diagrams associated with unsuppressed η' loops in the quenched approximation. They appear to be extremely small numerically for the light quark masses we consider[6], and are neglected in our baryon analysis. However, we do include the terms of order $m_q^{3/2}$. Thus a general octet baryon mass is written in an expansion of the form

$$m_B = A(e_{q1}, e_{q2}, e_{q3}) + \sum_i m_{qi} B_i(e_{q1}, e_{q2}, e_{q3}) + \sum_{i,j} (m_{qi} + m_{qj})^{3/2} C_{ij}(e_{q1}, e_{q2}, e_{q3}) \quad (1)$$

where e_{q1}, e_{q2}, e_{q3} are the three quark charges, and m_{q1}, m_{q2}, m_{q3} are the three bare quark masses, defined in terms of the Wilson hopping parameter by $(\kappa^{-1} - \kappa_c^{-1})/2a$. (Here a is the lattice spacing.) Each of the coefficients A, B_i, C_{ij} in (1) is then expanded in powers of the quark charges e_{q1}, e_{q2}, e_{q3}, with terms up to fourth order for A, second order for B_i, and with no charge dependence assumed for the nonanalytic C_{ij} terms. Because of the electromagnetic self-energy shift, the value of the critical hopping parameter must be determined independently for each quark charge. This is done [2] by requiring that the mass of the neutral pseudoscalar meson vanish at $\kappa = \kappa_c$. The results obtained in [2] for the κ_c values for various electric charge are inputs to our baryon calculations (as are the light quark masses extracted from the pseudoscalar meson spectrum). The fitting formula (1) turns out to have 30 parameters once all symmetries are exploited.

The success of the procedure outlined above clearly depends on the accurate extraction of the baryon masses for a large class of baryon states built from quarks of varying mass and electric charge. As in the meson studies of our previous work[2], quark propagators were calculated at hopping parameters (κ) 0.161, 0.165 and 0.1667 (with $\beta = 5.7$) for charge zero quarks. For the nonzero charge quarks, the hopping parameters were shifted by an amount computed from improved lattice perturbation theory in order to keep the quark masses for nonzero electric charge close to their values at zero charge[2]. A preliminary study using both single and multistate fits and employing local and smeared sources and sinks indicated that stable effective mass plots with reasonably small statistical errors could only be obtained

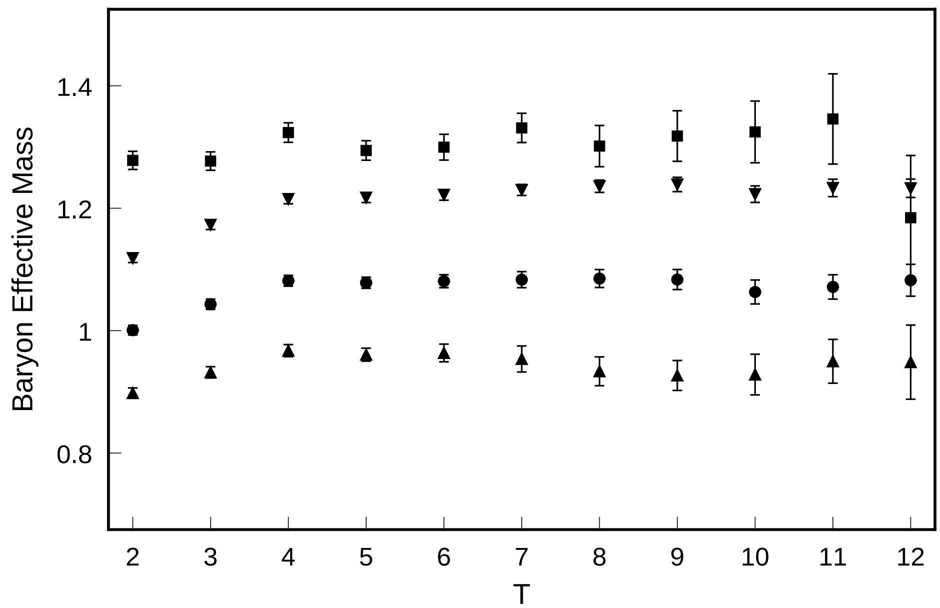

Figure 1: Some typical effective mass plots from smeared-local baryon correlators. Effective masses and times in lattice units.

from correlators with smeared sources and local sinks. A single quark propagator smearing function (obtained from a lattice relativistic quark model with parameters chosen to fit a pion wavefunction) was used throughout, for both the meson and baryon studies, so it is not surprising to find that the systematic errors induced by higher states were minimal in a limited range of baryon masses. In particular, baryons with masses (in lattice units) between 1.10 and 1.30 were found to give sizable effective mass plateaux with fairly small statistical errors (see Fig.1). The errors are obtained from single-elimination jackknife averages using 187 gauge configurations, each separated by 1000 Monte Carlo sweeps.

Octet baryon states correspond to a mixed spin-flavor symmetry, so correlators were computed for two distinct spin-flavor combinations. In the first combination, the first two quarks are coupled to spin and isospin 1 - as sigma hyperons can only be constructed in this way, this combination is called "sigma-type". In the second combination (dubbed "lambda-type"), the first two quarks are coupled to spin and isospin 0. Isospin-$\frac{1}{2}$ states (nucleon and Ξ) can be constructed from either the sigma or lambda type spin-flavor combinations. Although the isospin-$\frac{1}{2}$ masses determined from either combination must agree in the full theory in the continuum limit, quenched and finite lattice spacing corrections will lead to discrepancies in simulations on small lattices. Since we are only trying to determine the splittings within a given isomultiplet, we have chosen to determine the scale in all cases by fixing the average nucleon mass at its physical value. For the sigma-type combinations, our β=5.7 configurations (with unimproved action) then lead to a scale of about a^{-1}=1.37 GeV, while the scale for lambda-type combinations is found to be about a^{-1}=1.33 GeV.

As mentioned previously, we find the cleanest effective-mass plateaux in a limited range of baryon masses, presumably because of the fixed smearing wavefunction used for all the quark propagators. It should be emphasized that even a limited mass range for the baryons still contains baryons with widely ranging quark masses and charges, so the fitting formula (1) can be used to extract the A, B_i, C_{ij} coefficients, allowing an extrapolation to physical values of quark mass and electric charge. We have further restricted the portion of the baryon spectrum being fitted by varying the baryon mass window (for each choice of Euclidean time window used to extract a mass) until the χ^2/dof was minimal. For example, using a Euclidean time window from t =5 to t =8, the mass window (lattice units) from 1.20 to 1.26 was found to contain 74 sigma-type baryons. Determining the 30 parameters in (1) by fitting this set of masses gave a χ^2/dof of 1.33 . By contrast, using the mass window from 1.15 to 1.20 (122 baryons), the chi-square fit minimizes at χ^2/dof=2.16. For each choice of Euclidean time window, we have performed the fit to (1) using a baryon mass window which optimizes the χ^2/dof. For the lambda-type baryons, the best fit was obtained for the time window t =5 to t =9 using 54 baryons in the mass range 1.21 to 1.26, giving a χ^2/dof of 1.40.

The chi-squared fit to the chiral and electromagnetic expansion formula gives a central value for each fitting parameter, and a full 30x30 covariance matrix specifying the correlated errors on the fit parameters. With this information, one can determine the mass of any given octet baryon by extrapolating to physical values of quark mass and charge, together with an error which properly includes the statistical correlations among the fit parameters. The propagators for different electric charge are highly correlated, so it is not surprising to find that the statistical error on the center of gravity of baryon isomultiplets is considerably larger than the error on splittings within multiplets (typically, two orders of magnitude). This is apparent in Table 1, where the raw lattice results for the baryon octet spectrum with statistical errors, but no corrections for finite volume or quark-loop effects, are given for

Table 1: Raw lattice results for baryon octet (β=5.7, 12^3x24, 187 configurations). Quark masses are the bare lattice masses for Wilson action. All results are in MeV.

Baryon State	Window 5-8 (χ^2/dof=1.33)	Window 6-9 (χ^2/dof=1.51)	Window 5-9 (χ^2/dof=1.57)
Parameters $m_{u,d,s}$	$a^{-1} = 1370$ (3.57,7.10,155)	$a^{-1} = 1370$ (3.57,7.10,155)	$a^{-1} = 1280$ (3.88,7.54,180)
N	935.92±42.4	952.24 ± 43.6	942.13 ± 36.7
P	933.07 ±42.9	949.42 ± 44.0	940.00 ± 37.1
N-P	2.83 ± 0.56	2.82 ± 0.57	2.13 ± 0.50
Σ^+	1171.6 ± 25.6	1181.4 ± 26.1	1181.5 ± 19.2
Σ^0	1175.1 ± 25.3	1185.0 ± 25.8	1184.4 ± 18.9
Σ^-	1179.1 ± 25.0	1189.1 ± 25.5	1187.8 ± 18.7
$\Sigma^0 - \Sigma^+$	3.43 ± 0.39	3.56 ± 0.38	2.92 ± 0.34
$\Sigma^- - \Sigma^0$	4.04 ± 0.36	4.13 ± 0.43	3.36 ± 0.33
$\Sigma^+ + \Sigma^- - 2\Sigma^0$	0.61 ± 0.19	0.57 ± 0.26	0.44 ± 0.19
Ξ^-	1312.9 ± 14.5	1317.7 ± 15.9	1319.1 ± 10.9
Ξ^0	1308.2 ± 14.6	1312.7 ± 16.1	1314.8 ± 11.0
$\Xi^- - \Xi^0$	4.72 ± 0.24	5.01 ± 0.33	4.30 ± 0.24
Λ^0	1098 ± 52	1107 ± 22	1104 ± 69

three different choices of Euclidean time fitting window. Corrections due to these systematic effects will be estimated and included below.

In the second row of Table 1 we indicate the quark mass parameters and lattice scale assumed in generating the sigma-type masses for each of the fitting window choices. The lattice scale has been fixed in each case by requiring the nucleon center of gravity to sit at (roughly) the physical value. The masses within isomultiplets are highly correlated, so that the errors on the splittings are far smaller than on the multiplet center of gravity. This is expected since all the masses are calculated on the same set of gauge configurations. Once the lattice scale is fixed, we input up and down quark masses as previously determined [2] by analysis of the pseudoscalar meson spectrum (these will depend on the scale). The strange quark mass is known to fall at a higher value when determined from the baryon spectrum (a discrepancy which will presumably disappear on removal of finite a and quenched errors), so we have chosen to fix it using the center of gravity of the Ξ hyperon, which has the smallest statistical errors in our analysis. The center of gravity of the Σ multiplet and the Λ mass are then predictions of the analysis. The variations between the columns of Table 1 may be taken as an indication of the size of systematic errors associated with our choice of fitting window, which are generally smaller than the statistical errors. (Our bare quark masses correspond to an unimproved Wilson action at β=5.7, and are significantly larger than the continuum extrapolated $\bar{\rm M}$S values.)

For the remainder of this letter, we focus our attention on the splittings within isospin multiplets. Although the statistical errors are much smaller here, the relative impact of systematic errors (primarily those induced by finite volume, elimination of virtual quark loops, and lattice discretization) is far more important. To simplify the discussion, we shall henceforth restrict ourselves to the choice of Euclidean time-window 5-8 (which moreover gives the smallest χ^2), i.e. the second column of Table 1. By phenomenological estimates, discussed below, we find that the finite volume errors amount in all cases to shifts of less than 1 MeV, and are therefore totally irrelevant to the discussion of the isomultiplet centers of gravity. We have estimated and included these finite volume corrections (as well as quenched corrections to splittings) below- the corrections due to lattice discretization and finite volume will be studied directly in an upcoming run employing improved actions and larger lattices.

Given the presence of massless physical degrees of freedom we expect finite volume effects which fall only as inverse powers of the lattice size. These corrections can be studied directly on the lattice by repeating the calculations on lattices of varying physical volume. For the present, we estimate them by using the known dominance [8] of the Born contribution to the dispersive evaluation of the Cottingham [9] formula. The self-energy shift of a hadronic state arising from single photon exchange can be written as a sum of an electric and magnetic contribution, which on a spatial LxLxL lattice take the form

$$\delta m_{\rm el} = 2\pi\alpha m \frac{1}{L^3}\sum_{\vec{q}\neq 0}\frac{G_E(q)^2}{|q|}\{\frac{2}{q^2+4m^2}+\frac{1}{2m^2}(\sqrt{1+\frac{4m^2}{q^2}}-1)\} \quad (2)$$

$$\delta m_{\rm mag} = -\frac{\pi\alpha}{2m^3}\frac{1}{L^3}\sum_{\vec{q}\neq 0}|q|G_M(q)^2\{\sqrt{1+\frac{4m^2}{q^2}}-1-\frac{1}{2}\frac{1}{1+\frac{q^2}{4m^2}}\} \quad (3)$$

where the momentum vectors $\vec{q}$ are the appropriately discretized bosonic momenta for the finite LxLxL lattice. The elimination of the zero mode in the noncompact formulation [2] of the electromagnetic field we employ means that the $\vec{q}=0$ term is omitted in (2,3). The finite volume error can then be extracted by studying the dependence of (2,3) on L, holding the lattice spacing fixed. The charged baryon states contain a monopole contribution which falls as 1/L in the infinite volume limit. Using a dipole Ansatz [7] for the electric and magnetic form factors, one finds a positive shift $\delta m_{\rm el}$= 0.84 MeV for the proton in going from L=12 to L=∞. This electric shift is essentially the same for all charged members of the octet (a static approximation is very good for this quantity). In addition there is a magnetic finite volume shift of $\delta m_{\rm mag}$= -0.16 MeV for the proton, giving a total shift of 0.68 MeV. The finite volume corrections for the neutral states are much smaller- typically less than 0.1 MeV- and have a $1/L^3$ dependence in the infinite volume limit. Resonance and Regge region contributions are estimated to be no larger [8] than about 30% of the Born term, so there is possibly an error of the order of 0.2 MeV in this result (probably a conservative estimate, as the higher energy intermediate states correspond to shorter distance processes which should be less sensitive to finite volume effects). The finite volume shifts obtained in this way for each of the isospin splittings in the baryon octet are indicated in column 3 of Table 2, and our final estimate (including the finite volume correction as well as quenched error estimate- see below) for the baryon mass in column 5. As the L-dependence of these contributions is known, calculations on larger lattices will eventually allow a model-independent extrapolation to infinite volume.

Our calculation using quenched gauge configurations neglects graphs with internal quark loops. Such graphs are known [10, 3] to change the nonanalytic $m_q^{3/2}$ dependence in physical baryon masses, by introducing additional meson emission and reabsorption processes. Such processes also result in a nonnegligible shift in isospin splittings [7]. For example, in the static limit where the nucleon mass is infinite, the effect of the pion meson clouds surrounding the neutron and proton is to *decrease* the neutron-proton splitting by an amount (in the infinite volume limit) $0.43\Delta M_0$, where ΔM_0 is the nucleon splitting in the absence of a virtual pion cloud. This result is reduced slightly when the meson cloud contribution is evaluated fully relativistically- one then obtains $0.41\Delta M_0$. However the shift induced by the meson cloud turns out to be much more sensitive to finite volume effects. We shall use the static approximation [7] but include the effects of all octet pseudoscalar mesons (assuming SU(3) symmetry with a $d:(f+d)$ ratio of 0.62). Discretizing the second order shift formula on a

Table 2: Final results for baryon octet splittings (β=5.7, 12^3x24, 187 configurations). Errors on lattice results are statistical only. All results are in MeV.

Level Splitting	Raw Lattice	Finite Volume	Meson Cloud	Total Lattice	Physical Splitting
N - P	2.83 $\pm$ 0.56	-0.75	-0.53	1.55 $\pm$ 0.56	1.293
$\Sigma^0 - \Sigma^+$	3.43 $\pm$ 0.39	-0.80	-0.16	2.47 $\pm$ 0.39	3.18 $\pm$ 0.1
$\Sigma^- - \Sigma^0$	4.04 $\pm$ 0.36	+0.86	-0.27	4.63 $\pm$ 0.36	4.88 $\pm$ 0.1
$\Sigma^+ + \Sigma^- - 2\Sigma^0$	0.61 $\pm$ 0.19	+1.66	-0.11	2.16 $\pm$ 0.19	1.70 $\pm$ 0.15
$\Xi^- - \Xi^0$	4.72 $\pm$ 0.24	+0.86	+0.10	5.68 $\pm$ 0.24	6.4 $\pm$ 0.6

LxLxL lattice, one has

$$\delta m_{\rm mes} = -C\frac{g_A^2}{f_\pi^2}\frac{1}{L^3}\sum_{\vec{q}} G(q)^2\frac{\vec{q}^2}{2E(q)}\frac{1}{E(q)+M_2-M_1} \tag{4}$$

Here, C is an SU(3) Clebsch-Gordon coefficient for emission and reabsorption of the meson of mass m from a baryon of mass M_1, producing a virtual baryon of mass M_2. $E(q)$ is the virtual meson energy, and $G(q)$ is a dipole form-factor which we take to be $G(q) = \frac{1}{(1+q^2/0.71\mathrm{GeV}^2)^2}$. In general the meson cloud shift includes contributions from quenched nonplanar graphs in the cases where the emitted meson only contains valence quarks of the external baryon, so these estimates should only be regarded as a rough indication of the magnitude and sign (and probably, an overestimate), of the quenched correction. Setting L=12 and using a lattice scale a^{-1}=1370 MeV, together with the quenched masses from column 2 of Table 1, we obtain the meson cloud shifts given in column 4 of Table 2. The lattice results, corrected for finite volume and meson cloud effects, are given in column 5, and the physical values in column 6.

The results in Table 2 (which still need to be corrected for finite lattice spacing effects) suggest that the baryon isomultiplet splittings prefer a slightly larger up-down quark mass splitting than obtained from analysis of the pseudoscalar meson spectrum. Indeed the sigma and xi splittings could be increased by (say) 0.5 MeV without leading to serious disagreement with the nucleon splitting, given the large statistical error on the latter. Amusingly, the combination $\Sigma^+ + \Sigma^- - 2\Sigma^0$, in which linear quark mass effects cancel, is dominated by finite volume corrections with the present size lattice. It is evident that the extremely delicate level of baryon fine structure being considered here makes essential a detailed study of all systematic effects, with improved statistics on larger lattices. Such a study is in progress.

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